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Analyticity of Electromagnetic Fields in Regions Characterized by Analytic Dielectric Parameters and Analytic Sources

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***Abstract*—In this paper, the analyticity of time-harmonic electromagnetic fields in regions characterized by analytic dielectric parameters and analytic sources is proven.**

***Index Terms*—Electromagnetic theory, theoretical electromagnetics.**

I. Introduction

The knowledge of the analytic behavior of the electromagnetic field is of fundamental importance for the solutions of some theoretical electromagnetic problems—typically, uniqueness problems. Thus, for example, Müller [1] proved the uniqueness of the solutions of some boundary value problems (e.g., Theorem 34) and the uniqueness of the solutions of some scattering problems (e.g., Theorem 61) by using the analyticity of the electromagnetic field in any source-free and homogeneous region. Moreover, Caorsi and Raffetto [2] proved an extension of the classical uniqueness theorem [3] for time-harmonic electromagnetic boundary value problems by using a result (proved by Müller) based again on the analyticity of the electromagnetic field in any source-free and homogeneous region.

It is important to note that the indicated applications of the analyticity of the electromagnetic field are restricted to homogeneous dielectric materials. However, many problems of physical interest, such as the spherical Luneberg lens problem [4], [5], or the study of the propagation along the old graded index optical fibers, or the reflection of electromagnetic waves from continuously stratified media [6], or even the theory of Maxwell's "fish-eyes" lens [4], involve materials with continuously varying dielectric properties.

Consequently, it would be important to generalize the indicated result of analyticity to "analytically inhomogeneous" materials, i.e., to materials having (nonconstant) analytic dielectric parameters. For example, this generalization could allow further extensions of the uniqueness theorem for time-harmonic electromagnetic boundary value problems (i.e., to cases involving dielectric materials which are lossy in a part of the region of interest and "analytically inhomogeneous" and lossless in the rest of the region).

The goal of this paper is to move toward such a generalization of the analytic behavior of the electromagnetic field. In particular, it will be shown that the electromagnetic field is analytic in any open region characterized by analytic dielectric parameters and analytic sources.

II. A Result on the Analyticity of the Electromagnetic Field

In this section, we will prove our main result about the analyticity of the electromagnetic field. It is important to note that in this paper a scalar or vector field is called analytic in an open region $\Omega \subset R^3$ if it is defined in Ω and if it can be developed in multiple power series in a neighborhood of every point belonging to Ω [7, p. 212], [8, p. 170]. Then, in particular, by "analyticity of the electromagnetic field in Ω," we mean that the electric and magnetic fields can be developed in multiple power series in a neighborhood of every point belonging to Ω.

Theorem Let Ω be an open region in R^3. Moreover, let $\mathbf{E}$ and $\mathbf{H}$ be twice continuously differentiable vector fields in Ω (i.e., $\mathbf{E} \in [C^2(\Omega)]^3$ and $\mathbf{H} \in [C^2(\Omega)]^3$), such that

$$\begin{cases} \nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu(\mathbf{r})\mathbf{H}(\mathbf{r}), & \text{in } \Omega \\ \nabla \times \mathbf{H}(\mathbf{r}) = \mathbf{J}(\mathbf{r}) + j\omega\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}), & \text{in } \Omega \end{cases} \tag{1}$$

where ω is the angular frequency (assumed to be strictly positive), $\varepsilon(\mathbf{r})$ and $\mu(\mathbf{r})$ are complex scalar fields analytic in Ω, and $\mathbf{J}(\mathbf{r})$, which represents the source of the electromagnetic field, is a complex vector field analytic in Ω.

Then $\mathbf{E}$ and $\mathbf{H}$ are analytic vector fields in Ω.

Proof: By combining both equations appearing in (1), we obtain

$$\begin{aligned} \nabla \times \left[\frac{1}{\mu(\mathbf{r})}\nabla \times \mathbf{E}(\mathbf{r})\right] &= -j\omega\nabla \times \mathbf{H}(\mathbf{r}) \\ &= -j\omega\mathbf{J}(\mathbf{r}) + \omega^2\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}), \qquad \text{in } \Omega. \end{aligned} \tag{2}$$

By using the vector identity

$$\nabla \times (u\mathbf{V}) = \nabla u \times \mathbf{V} + u\nabla \times \mathbf{V} \tag{3}$$

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we can obtain an equivalent expression for the left-hand side (LHS) term (it is important to note that the indicated transformation is valid as a consequence of the hypothesis $\mathbf{E} \in [C^2(\Omega)]^3$):

$$\nabla \left[\frac{1}{\mu(\mathbf{r})}\right] \times [\nabla \times \mathbf{E}(\mathbf{r})] + \frac{1}{\mu(\mathbf{r})} \nabla \times [\nabla \times \mathbf{E}(\mathbf{r})]. \tag{4}$$

By virtue of the other vector identity

$$\nabla \times (\nabla \times \mathbf{V}) = \nabla(\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V} \tag{5}$$

the LHS term of (2) can be written as follows:

$$\nabla \left[\frac{1}{\mu(\mathbf{r})}\right] \times [\nabla \times \mathbf{E}(\mathbf{r})] + \frac{1}{\mu(\mathbf{r})} \nabla[\nabla \cdot \mathbf{E}(\mathbf{r})] - \frac{1}{\mu(\mathbf{r})} \nabla^2 \mathbf{E}(\mathbf{r}) \tag{6}$$

and (2) becomes

$$\nabla \left[\frac{1}{\mu(\mathbf{r})}\right] \times [\nabla \times \mathbf{E}(\mathbf{r})] + \frac{1}{\mu(\mathbf{r})} \nabla[\nabla \cdot \mathbf{E}(\mathbf{r})] - \frac{1}{\mu(\mathbf{r})} \nabla^2 \mathbf{E}(\mathbf{r}) = -j\omega \mathbf{J}(\mathbf{r}) + \omega^2 \varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}), \qquad \text{in } \Omega. \tag{7}$$

By using the well-known property

$$\nabla \cdot (\nabla \times \mathbf{V}) = 0 \tag{8}$$

for any twice continuously differentiable vector field ($\mathbf{H} \in [C^2(\Omega)]^3$ is important in this case), we obtain the following from (1):

$$j\omega \nabla \cdot [\varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r})] = -\nabla \cdot \mathbf{J}(\mathbf{r}) \tag{9}$$

and, by virtue of the vector identity

$$\nabla \cdot (u\mathbf{V}) = \nabla u \cdot \mathbf{V} + u \nabla \cdot \mathbf{V} \tag{10}$$

we obtain

$$j\omega \mathbf{E}(\mathbf{r}) \cdot \nabla \varepsilon(\mathbf{r}) + j\omega \varepsilon(\mathbf{r}) \nabla \cdot \mathbf{E}(\mathbf{r}) = -\nabla \cdot \mathbf{J}(\mathbf{r}) \tag{11}$$

which is equivalent to

$$\nabla \cdot \mathbf{E}(\mathbf{r}) = \frac{-j\omega \mathbf{E}(\mathbf{r}) \cdot \nabla \varepsilon(\mathbf{r}) - \nabla \cdot \mathbf{J}(\mathbf{r})}{j\omega \varepsilon(\mathbf{r})} \tag{12}$$

where the denominator is different from zero, since ω has been assumed to be strictly positive and at least the real part of $\varepsilon(\mathbf{r})$ is strictly positive.

By substituting the right-hand side (RHS) term of (12) for the divergence of the electric field in (7), we obtain

$$\nabla \left[\frac{1}{\mu(\mathbf{r})}\right] \times [\nabla \times \mathbf{E}(\mathbf{r})] + \frac{1}{\mu(\mathbf{r})} \nabla \left[\frac{-j\omega \mathbf{E}(\mathbf{r}) \cdot \nabla \varepsilon(\mathbf{r}) - \nabla \cdot \mathbf{J}(\mathbf{r})}{j\omega \varepsilon(\mathbf{r})}\right] - \frac{1}{\mu(\mathbf{r})} \nabla^2 \mathbf{E}(\mathbf{r}) = -j\omega \mathbf{J}(\mathbf{r}) + \omega^2 \varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}), \qquad \text{in } \Omega. \tag{13}$$

By moving all the unknown terms to the LHS and all the "source" terms to the RHS, we obtain

$$\nabla^2 \mathbf{E}(\mathbf{r}) + \nabla \left[\frac{\mathbf{E}(\mathbf{r}) \cdot \nabla \varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})}\right] + \mu(\mathbf{r}) [\nabla \times \mathbf{E}(\mathbf{r})] \times \nabla \left[\frac{1}{\mu(\mathbf{r})}\right] + \omega^2 \varepsilon(\mathbf{r}) \mu(\mathbf{r}) \mathbf{E}(\mathbf{r}) = j\omega \mu(\mathbf{r}) \mathbf{J}(\mathbf{r}) - \nabla \left[\frac{\nabla \cdot \mathbf{J}(\mathbf{r})}{j\omega \varepsilon(\mathbf{r})}\right], \qquad \text{in } \Omega. \tag{14}$$

This is a linear system of second-order partial differential equations. Systems of partial differential equations are classified by type, according to the properties of the characteristic determinant [9]. This is, for the particular case of system (14), the determinant of a 3 × 3 matrix, A, whose element in the ith row and jth column, A_{ij}, is given by the sum of the coefficients of the second-order derivatives $\partial^2 E_j / \partial x_l \partial x_m$, $l, m = 1, 2, 3$, in the ith equation, multiplied by the real second-order polynomials $\lambda_l \lambda_m$, where λ_1, λ_2, and λ_3 are real parameters [9]. Thus for example, assuming a system of Cartesian coordinates, i.e., $x_1 = x$, $x_2 = y$, $x_3 = z$, $E_1 = E_x$, $E_2 = E_y$, and $E_3 = E_z$, system (14) is characterized by

$$A_{11} = \sum_{l=1}^{3} \sum_{m=1}^{3} a_{lm} \lambda_l \lambda_m \tag{15}$$

where a_{lm} is the coefficient of $\partial^2 E_1 / \partial x_l \partial x_m$ in the first equation of system (14). Then

$$A_{11} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2. \tag{16}$$

Analogously,

$$A_{22} = A_{33} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \tag{17}$$

and

$$A_{12} = A_{13} = A_{21} = A_{23} = A_{31} = A_{32} = 0 \tag{18}$$

i.e.,

$$A = \begin{bmatrix} \lambda_1^2 + \lambda_2^2 + \lambda_3^2 & 0 & 0 \\ 0 & \lambda_1^2 + \lambda_2^2 + \lambda_3^2 & 0 \\ 0 & 0 & \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \end{bmatrix} \tag{19}$$

and the characteristic determinant is

$$k(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^3. \tag{20}$$

According to [9], system (14) is called uniformly elliptic in the domain Ω if we can indicate nonzero constants k_0 and k_1 of the same sign, such that

$$k_0(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^3 \leq k(\lambda_1, \lambda_2, \lambda_3) \leq k_1(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^3 \tag{21}$$

everywhere in Ω.

By using (20), we can obviously conclude that system (14) is uniformly elliptic in Ω and in particular, elliptic in Ω [9] (see also [7]). Moreover, as a consequence of the hypotheses on the analyticity in Ω of the dielectric parameters and sources, (14) is characterized by analytic coefficients (in Ω) and by an analytic known term (in Ω). Then, according to [7], any twice continuously differentiable solution of (14) is analytic in Ω, i.e., $\mathbf{E}$ is analytic in Ω.

By using an analogous procedure, we could also prove the analyticity of the magnetic field. However, this result can also be obtained by using

$$\mathbf{H}(\mathbf{r}) = \frac{\nabla \times \mathbf{E}(\mathbf{r})}{-j\omega \mu(\mathbf{r})} \tag{22}$$

and the analyticity of $\mathbf{E}$ and μ in Ω (having assumed $\omega > 0$).

As a final remark, it could be important to note that $\mathbf{J}(\mathbf{r}) = 0 \; \forall \mathbf{r} \in \Omega$ is a vector-field analytic in Ω. Then, the theorem also holds in this case and, consequently, we can conclude that any twice continuously differentiable solution of the source-free problem

$$\begin{cases} \nabla \times \mathbf{E}(\mathbf{r}) = -j\omega \mu(\mathbf{r}) \mathbf{H}(\mathbf{r}), & \text{in } \Omega \\ \nabla \times \mathbf{H}(\mathbf{r}) = j\omega \varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}), & \text{in } \Omega \end{cases} \tag{23}$$

is analytic in Ω.

III. Conclusions

A proof of the analytic behavior of time-harmonic electromagnetic fields in open regions characterized by analytic dielectric parameters and analytic sources has been presented. This result can be of some importance for theoretical questions in electromagnetics, typically in proving uniqueness theorems.

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Effect of Finite Metallization and Inhomogeneous Dopings on Slow-Wave-Mode Propagation

Jakub J. Kucera and Ronald J. Gutmann

***Abstract*— A finite-element simulation has been implemented to evaluate the slow-wave-mode propagation characteristics in metal–insulator–semiconductor (MIS) waveguiding structures. Particular emphasis has been placed on coplanar waveguides compatible with silicon integrated circuits (IC’s), with an objective of evaluating the effect of inhomogeneous doping on propagation characteristics. The simulator has been successfully benchmarked against a number of cases presented in the literature, including MIS coplanar waveguides. The effect of inhomogeneous doping and finite metallization in maintaining a large slowing factor while reducing the attenuation constant and increasing transmission-line Q is presented, and constraints on slow-wave-mode passive components are discussed.**

***Index Terms*—Coplanar waveguides, slow-wave mode.**

I. Introduction

There is continued interest in slow-wave-mode propagation in silicon integrated circuits (IC’s), both in preventing such propagation in digital IC’s and in utilizing such propagation in microwave-analog IC’s for passive components. In the latter application, maintaining a high slowing factor and achieving a low attenuation factor are critical. Work to date indicates that adjusting dimensional and electrical parameters with uniform semiconductor doping does not result in propagation characteristics useful for passive components [1]–[4], although simulation results obtained with nonuniform doping profiles indicate that more attractive characteristics can be obtained [5], [6].

In this paper, a two-dimensional (2-D) electromagnetic simulator is developed to determine the propagation characteristics of metal–insulator–semiconductor (MIS) structures with arbitrarily doped substrates based upon the finite-element method (FEM). Since the propagating modes within an inhomogeneous structure are hybrid, quasi-static approaches can only be used in limited cases. In particular, when dealing with arbitrary substrate dopings, a quasi-static approach is insufficient and a full-wave analysis is required. With our FEM simulator, the advantages of lateral doping profiles on propagation characteristics of coplanar waveguide (CPW) structures have been evaluated. The influence of line parameters such as the finite metallization on the propagation characteristics of the slow-wave mode is presented, and upper bounds of achievable transmission-line quality factors are discussed.

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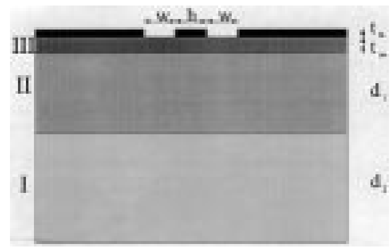
TABLE I
Percentage of the Conductor Loss to the Overall Loss for a MIS Coplanar Waveguide ($w = 50$ μm, $t_{ox} = 0.1$ μm, $d_1 = 150$ μm, $d_2 = 660$ μm, $\sigma = 3.7$ S/mm, $t_m = 2$ μm, and $\sigma_m = 2.7 \cdot 10^4$ S/mm)

h [μm]	@ 1 GHz	@ 10 GHz
1	99.5	83.5
5	97.6	49
100	57.3	2.2



II. Effect of Finite Metallization and Inhomogeneous Dopings

The effect of quantities such as center strip or slot width, oxide thickness, and substrate resistivity on the slow-wave-mode propagation in MIS CPW’s has been extensively studies [3], [5], [7], but the effect of imperfect conductors generally has been neglected. While the finite thickness of the metallization has a minor effect, the conductivity is of great importance. It is known that with decreasing linewidth, the metal conductor losses increase and can constitute the dominant loss mechanism [4]. In the lower gigahertz range, the slowing factor is also affected to a large extent since the current penetrates deep into the metal surface so that the metal behaves like a very lossy dielectric. The surface-impedance approach based on the skin depth becomes questionable, because the effect of the lossy metal on the slowing factor is ignored and the current is not necessarily confined to the surface of the conductor. In our FEM approach the metal losses and the field penetration within the conductor are precisely handled by treating the metal layers as lossy dielectrics with a dielectric constant of unity. We estimate the contribution of the metal losses to the overall losses for a particular waveguide by calculating the attenuation for both a perfect and a lossy metal conductor. The metal losses dominate the overall losses at 1 GHz even for a wide center strip (100 μm), and even at 10 GHz they cannot be neglected for narrow strips, as shown in Table I.